Quartiles and Interquartile Range (IQR)

Methods of Quartile Calculation

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Learning Objectives

- Understand quartiles and the interquartile range (IQR)
- Distinguish between odd and even number of observations
- Learn two common methods of calculating quartiles
- Apply quartile calculations to numerical examples

What are Quartiles?

Quartiles divide ordered data into four equal parts.

0%	25%	50%	75%
	Q_1	Q_2	Q_3

- Q_1 : First Quartile (25th percentile)
- Q_2 : Median (50th percentile)
- Q_3 : Third Quartile (75th percentile)

Interquartile Range (IQR)

The Interquartile Range measures the spread of the middle 50% of the data.

$$\boxed{\text{IQR} = Q_3 - Q_1}$$

Advantages

- Resistant to outliers
- Better measure of spread than range for skewed data

Outlier Detection

$$\text{Lower Limit} = Q_1 - 1.5(\text{IQR})$$

$$\text{Upper Limit} = Q_3 + 1.5(\text{IQR})$$

Method 1: Median-of-Halves Method

Procedure

1. Arrange data in ascending order
2. Find the median (Q_2)
3. Divide data into lower and upper halves
4. Find medians of both halves

Important Difference

Odd Number of Observations

The median is excluded before finding Q_1 and Q_3

Even Number of Observations

Data splits equally into two halves

Even Number of Observations

Example:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10$$

$$n = 10 \quad \text{(even)}$$

Step 1: Find Median

$$Q_2 = \frac{5+6}{2} = 5.5$$

Step 2: Divide into Halves

Lower half:

$$1, 2, 3, 4, 5$$

Upper half:

$$6, 7, 8, 9, 10$$

Even Number of Observations: Quartiles

Lower half:

$$1, 2, 3, 4, 5$$

Median of lower half:

$$Q_1 = 3$$

Upper half:

$$6, 7, 8, 9, 10$$

Median of upper half:

$$Q_3 = 8$$

$$\boxed{\text{IQR} = 8 - 3 = 5}$$

Odd Number of Observations

Example:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$$

$$n = 11 \quad \text{(odd)}$$

Step 1: Find Median

$$Q_2 = 6$$

Important: Exclude the Median

Lower half:

$$1, 2, 3, 4, 5$$

Upper half:

$$7, 8, 9, 10, 11$$

Odd Number of Observations: Quartiles

Lower half:

$$1, 2, 3, 4, 5$$

Median:

$$Q_1 = 3$$

Upper half:

$$7, 8, 9, 10, 11$$

Median:

$$Q_3 = 9$$

$$\boxed{\text{IQR} = 9 - 3 = 6}$$

Method 2: Percentile Position Formula

Quartile positions are calculated using:

$$Q_1 : \frac{n+1}{4}$$

$$Q_2 : \frac{n+1}{2}$$

$$Q_3 : \frac{3(n+1)}{4}$$

Interpretation

- Integer position $\rightarrow$ exact observation
- Decimal position $\rightarrow$ interpolation required

Interpolation

Suppose the quartile position is:

$$3.25$$

This means:

- Start at the 3rd observation
- Move 25% toward the 4th observation

Formula:

$$Q = x_i + f(x_{i+1} - x_i)$$

where:

- x_i = lower observation
- f = fractional part

Example Using Percentile Formula

Data:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$$

$$n = 12$$

First Quartile Position

$$\frac{12 + 1}{4} = 3.25$$

$$Q_1 = 3 + 0.25(4 - 3) = 3.25$$

Continuation of Example

Median Position

$$\frac{12+1}{2} = 6.5$$

$$Q_2 = 6 + 0.5(7-6) = 6.5$$

Third Quartile Position

$$\frac{3(12+1)}{4} = 9.75$$

$$Q_3 = 9 + 0.75(10-9) = 9.75$$

$$\boxed{\text{IQR} = 9.75 - 3.25 = 6.5}$$

Comparison of Methods

Method	Advantages	Limitations
Median-of-Halves	Simple and intuitive	May differ from software results
Percentile Formula	More precise interpolation	Slightly more computational work

Important Note

- Different textbooks and software may use different quartile definitions
- Always specify the method used

Key Takeaways

- Quartiles divide data into four equal parts
- Q_2 is the median
- IQR measures the spread of the middle 50% of data
- Odd and even sample sizes are handled differently
- Multiple quartile calculation methods exist

Thank You